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Operational momentum for magnitude ordering in preschool children and adults

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ABSTRACT

When adding or subtracting quantities, adults tend to overestimate addition outcomes and underestimate subtraction outcomes. They also shift visuospatial attention to the right when adding and to the left when subtracting. These *operational momentum* phenomena are thought to reflect an underlying representation in which small magnitudes are associated with the left side of space and large magnitudes with the right side of space. Currently, there is limited research on operational momentum in early childhood or for operations other than addition and subtraction. The current study tested whether English-speaking 3- and 4-year-old children and college-aged adults exhibit operational momentum when ordering quantities. Participants were presented with two experimental blocks. In one block of trials, they were tasked with choosing the same quantity they had previously seen three times; in the other block, they were asked to generate the next quantity in a doubling sequence composed of three ascending quantities. A bias to shift attention to the right after an ascending operation was found in both age groups, and a bias to overestimate the next sequential quantity during an ascending ordering operation was found in adults under conditions of uncertainty. These data suggest that, for children, the spatial biases during operating are more pronounced than the mis-estimation biases. These findings highlight the spatial underpinnings of operational momentum and suggest

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that both very young children and adults conceptualize quantity along a horizontal continuum during ordering operations, even before formal schooling.

Introduction

Adults (Cordes, Gelman, Gallistel, & Whalen, 2001), children (Halberda & Feigenson, 2008), preverbal infants (VanMarle & Wynn, 2006; Xu & Spelke, 2000), and nonhuman animals (Cantlon & Brannon, 2007; Garland, Low, & Burns, 2012) perceive and process nonsymbolic quantities in an approximate fashion. This large-number system yields inexact nonverbal representations of a given numerical magnitude as defined by Weber's law: The ease with which two quantities are discriminable is proportional to their ratio, not their absolute difference (e.g., 10 is more discriminable from 4 than 110 is from 104 despite an identical absolute difference; Cordes et al., 2001; Whalen, Gallistel, & Gelman, 1999).

Numerical magnitudes are represented on a spatially organized horizontal continuum known as the mental number line (Dehaene, 1992; Moyer & Landauer, 1967). For many people, this mental number line is structured such that small values are mentally placed to one side of space and large values to the other side of space. This predisposed lateralization is driven by evolved neurological architecture (Vallortigara, 2017). For instance, both rhesus macaques (*Macaca mulatta*) and newborn chicks exhibit a propensity to map number onto space from left to right asymmetrically (Drucker & Brannon, 2014; Rugani, Kelly, Szelest, Regolin, & Vallortigara, 2010). Further evidence for predisposed spatial–numerical associations comes from studies with human neonates (de Hevia, Veggiotti, Streri, & Bonn, 2017) and preverbal infants (Bulf, de Hevia, & Macchi Cassia, 2016; de Hevia, Izard, Coubart, Spelke, & Streri, 2014), which also illustrate small-left and large-right mappings. This lateralization is, for many populations, strengthened across the lifespan, namely among participants within Westernized cultures whose reading and writing direction is from left to right (Shaki, Fischer, & Göbel, 2012; see McCrink & Opfer, 2014, for a review). These culturally influenced processes are formed in early childhood and can reinforce the structure of spatial associations such as the mental number line (Dobel, Diesendruck, & Bolte, 2007).

Spatial associations between space and magnitude are recruited not only when discriminating two magnitudes but also when operating over these magnitudes, resulting in a phenomenon known as *operational momentum*. According to one popular account of this phenomenon, the asymmetrically oriented spatial associations lead to arithmetic miscalculations and spatial shifts of attention when operating over sets of objects (Knops, Viarouge, & Dehaene, 2009; Knops, Zitzmann, & McCrink, 2013; McCrink, Dehaene, & Dehaene-Lambertz, 2007; cf. Chen and Verguts (2012) and Prather (2012) for alternate nonspatial, accounts). Operational momentum is related to the perceptual phenomenon known as "representational momentum," in which participants have a tendency to anticipate the final location of a perceived moving target erroneously, displaced in the direction of target motion in dimensions of space, action, and/or the pitch of sounds (Ashida, 2004; Freyd & Finke, 1984; Freyd, Kelly, & DeKay, 1990; Getzmann, Lewald, & Guski, 2004). Just as moving objects have physical thrust, mental calculations exhibit a momentum of their own.

When mentally adding, subtracting, multiplying, and dividing, people experience a forward displacement along their mental representation in the direction of the operation's outcome (i.e., toward large numbers for addition and multiplication problems and toward small numbers for subtraction and division problems; Katz & Knops, 2014; McCrink et al., 2007). Thus, operational momentum is thought to be driven by attentional shifts along an internally represented horizontal continuum. In line with this view, Knops, Thirion, Hubbard, Michel, and Dehaene (2009) found evidence that the neural activity associated with rightward saccades is the same as that activated when viewing centrally presented addition problems, suggesting that addition shifts attention toward the right side of the mental number line. Furthermore, Knops, Viarouge, et al. (2009) found that during nonsymbolic addition adults preferentially select outcomes from the right side of a screen, whereas during subtraction they are biased toward the left side of the screen. Operational momentum is proposed to be the result of a type of logical heuristic (McCrink & Hubbard, 2017; McCrink & Wynn, 2009), in which participants generate a rule of "if adding, more" and "if subtracting, less," which is implemented via spatial shifts of attention. As with other heuristics (Kahneman & Tversky, 2013), operational momentum increases under conditions of estimation uncertainty, time pressure, and distraction (Knops, Viarouge, et al., 2009; McCrink & Hubbard, 2017; McCrink et al., 2007).

Operational momentum for addition and subtraction has also been documented in infants (McCrink & Wynn, 2009), a population found to have a left-to-right-oriented spatial–numerical mapping (Bulf et al., 2016; de Hevia, Girelli, Addabbo, & Macchi Cassia, 2014). Infants who see 4 objects added to a set of 6 objects look reliably longer to an incorrect outcome of 5 (an underestimation) than of 20 (an overestimation) (McCrink & Wynn, 2009). Thus, infants appear to also experience the mis-estimation aspect of operational momentum. This suggests that there may be an underlying common mechanism of shifting visuospatial attention (Bulf et al., 2016) while operating over magnitudes for both infants and adults (although to date no one has looked directly at spatial effects of operational momentum during adding and subtracting in infancy). Despite similarities in mis-estimation among infants and adults, there is not strong evidence for consistent overestimation and underestimation in childhood. For example, Knops et al. (2013) found that 6- and 7-year-olds display either no operational momentum effect or even an inverse effect in which they significantly overestimate subtraction problems rather than addition problems.

Given what is known about operational momentum for adding and subtracting, the effect should only be strengthened during the operation of ordering. Ordering underlies arithmetic reasoning because it is the ability to recognize that, in an ordered numerical series, one number is greater than another number that allows participants to transform numerical quantities and anticipate outcomes as they operate (Gallistel & Gelman, 1992). The process of ordering information is especially evocative of lateralized spatial structuring; in adults, any well-learned ordinal sequence—such as the days of the week (Gevers, Reynvoet, & Fias, 2004), months of the year (Gevers, Reynvoet, & Fias, 2003), or even a newly learned list of unrelated words (Previtali, de Hevia, & Girelli, 2010)—becomes represented on a horizontal continuum with initial information on one side of space and final information on the other side of space. Importantly, such mapping of ordinal information into a directional space is functional prior to the acquisition of symbolic knowledge and language given that Italian 7-month-old infants learn rule-like patterns from visual sequences when the sequences are presented from left to right but not when they are presented from right to left (Bulf, de Hevia, Gariboldi, & Macchi Cassia, 2017). Therefore, preverbal infants, like adults, organize not only numerical order along a directional space (de Hevia, Izard, et al., 2014) but also any kind of ordinal information (Bulf et al., 2017).

In particular, when it comes to ordering operations over magnitude, infants show evidence of operational momentum. Macchi Cassia, McCrink, de Hevia, Gariboldi, and Bulf (2016) and Macchi Cassia, Bulf, McCrink, and de Hevia (2017) showed that, when 4- and 12-month-old infants are habituated to a set of objects that progressively increases or decreases in number (Macchi Cassia et al., 2017) or physical size (Macchi Cassia et al., 2016) and are subsequently presented with ordinal sequences whose magnitude is larger or smaller compared with the initial sequence, they look longer at sequences in which the direction violates the operational momentum experienced during habituation (the smaller sequence after viewing ascension and the larger sequence after viewing descension).

Although these findings in infancy indicate that operational momentum may reflect an underlying component of an early system of arithmetic transformations, the limited evidence for operational momentum in toddlerhood and early childhood casts doubt as to whether there is developmental continuity for this mechanism. In the current study, the presence of operational momentum was tested during an ordering operation in English-speaking 3- and 4-year-old children and college-aged adults. In this experiment, participants were presented with an ascending sequence of magnitudes (Order trials) or a series of identical magnitudes (No Order trials) and were asked to generate the next answer in the series by selecting a response from two laterally presented arrays. The correct answer was presented alongside overestimates and underestimates of the correct outcome. In some cases, no correct answer was presented and participants needed to choose from two incorrect amounts. If participants experienced both the arithmetic and spatial components of operational momentum when ordering amounts, two overall patterns should emerge. First, when participants perform an ascending operation, they will choose the overestimated outcome more often than if they did not just perform an ascending operation. Second, on trials in which participants perform an ascending operation, they will be more likely to choose the right-side array (e.g., the side associated with "more" on the mental number line) compared with trials in which they must simply estimate an amount. Both of these effects were predicted to increase during trials in which no correct answer is present due to greater use of heuristics under conditions of uncertainty.

Method

Participants

In total, 50 preschoolers (M_{age} = 3.9 years, range = 3 years 0 months to 4 years 11 months) and 49 college-aged students participated in either a children's museum (children) or a university laboratory (adults) in a major urban area. An additional 8 children participated but were excluded from the final sample due to an unwillingness to complete the experiment. An additional 4 adults participated but were excluded from the final sample due to right-to-left scripted language fluency (n = 3) or computer error (n = 1). Sample size was determined using effect sizes from previous research examining preschool space–number relations, which are in the moderate range (effect size f = .18, with power [1 − ß err probability] = .95 and α = .05; McCrink, Shaki, & Berkowitz, 2014; Patro & Haman, 2012), for a main experimental design for each population that has two within-participants factors (block type and trial type) and one between-participants factor (gender). Gender was included in the power analysis pursuant to a National Institutes of Health (NIH) request that gender be included in all NIH-funded research designs, analyses, and reporting (Notice No. 15-102). In addition, there are some findings that suggest male individuals may have a stronger linear representation of number (Bull, Cleland, & Mitchell, 2013; Hutchison, Lyons, & Ansari, 2018), which may potentially result in a larger operational momentum effect for male individuals relative to female ones.

Design

Each participant received two experimental blocks of trials. Each block was preceded by 5 training trials to ensure that the participants understood the task. In the No Order block (8 trials), participants were tasked with choosing a numerosity that matched an identical number of objects shown for three preceding priming slides (i.e., a non-ordered sequence). In the Order block (18 trials), participants were tasked with choosing the numerosity that would arithmetically come next after viewing three priming slides of dots that doubled in numerosity (e.g., 4, then 8, then 16). After the presentation of these sequences, participants were shown two choice options: one on the left side and one on the right side of the screen. These choice options were (a) correct, (b) an overestimate of the correct answer, or (c) an underestimate of the correct answer. There were four test trial types (correct vs. overestimate, correct vs. underestimate, overestimate vs. underestimate, and tie trials). In the Order block there were three different versions of tie trials (underestimate vs. underestimate, correct vs. correct, and overestimate vs. overestimate), and in the No Order block there was only one version (correct vs. correct). This design allowed us to have the same magnitudes, on average, across the blocks but to keep the overall number of trials down for the children. Using this design, we could calculate the prevalence of overestimation (choice of overestimates in the correct vs. overestimate trials and overestimate vs. underestimate trials and choice of correct in the correct vs. underestimate trials) and the prevalence of right-side bias (choice of the right-side test array for the tie trials). Tables 1–4 provide the exact values used, and the side of the screen on which they were displayed, for both the training and testing trials. The order of blocks was counterbalanced between participants, and the presentation of trials within each block was randomized. There were fewer No Order trials than Order trials; although an earlier design had identical amounts of trials for both blocks, initial piloting revealed that the younger preschoolers were unable to tolerate more than approximately 24 trials plus the training for each block. Thus, the side of presentation for the correct answer in the No Order block was counterbalanced between trial types and in the Order block was counterbalanced both within and between trial types.

Table 1
Experimental trials with priming 4–8–16.

Trial type	Left-side probe	Right-side probe
Correct vs. underestimate	20	32[a]
	32[a]	20
Correct vs. overestimate	52[a]	32
	32	52[a]
Underestimate vs. overestimate	20	52[a]
	52[a]	20
Underestimate vs. underestimate (tie)	20	20[a]
Correct vs. correct (tie)	32	32[a]
Overestimate vs. overestimate (tie)	52	52[a]

[a] Test option that accords with the direction of operational momentum (either spatially or arithmetically) for an ascending operation.

Table 2
Experimental trials for Order block with priming 12–24–48.

Trial type	Left-side probe	Right-side probe
Correct vs. underestimate	60	96[a]
	96[a]	60
Correct vs. overestimate	154[a]	96
	96	154[a]
Underestimate vs. overestimate	60	154[a]
	154[a]	60
Underestimate vs. underestimate (tie)	60	60[a]
Correct vs. correct (tie)	96	96[a]
Overestimate vs. overestimate (tie)	154	154[a]

[a] Test option that accords with the direction of operational momentum (either spatially or arithmetically) for an ascending operation.

Table 3
Experimental trials for No Order block with priming 32–32–32.

Trial type	Left-side probe	Right-side probe
Correct vs. underestimate	20	32
Correct vs. overestimate	32	52
Underestimate vs. overestimate	52	20
Correct vs. correct (tie)	32	32

Table 4
Experimental trials for No Order block with priming 96–96–96.

Trial type	Left-side probe	Right-side probe
Correct vs. underestimate	96	60
Correct vs. overestimate	154	96
Underestimate vs. overestimate	60	154
Correct vs. correct (tie)	96	96

Stimuli and procedure

All stimuli consisted of dots .40 cm in diameter presented in a black frame that was 6.1 by 6.1 cm during priming and during the choice slide. The visual stimuli were created on Keynote presentation software and presented with SuperLab 5 on a 13-inch Hewlett Packard touchscreen laptop computer.

No order block

Training trials. Participants received 5 training trials, each consisting of three priming slides (a non-ordered sequence) presented by the experimenter and then two choice arrays in which a correct answer was present. The first trial was a demonstration performed by the experimenter in order to provide a background story and orient each participant to the computer touchscreen. In this trial, the experimenter presented two cartoon dancers inside a black square frame (the "dance floor"). As the experimenter flipped to the next slide, which showed the same two cartoon dancers but this time in a new spatial layout within the black square frame, she explained that the two dancers were still dancing. The third slide again showed two dancers *still* dancing. The experimenter then posed a question to the participant: "If our dancers are still dancing, which dance floor do you think will be ours?" The experimenter presented a screen with two choices and used the talk-aloud strategy to acclimate the participant to the logic behind her choice, explaining that she would touch the dance floor she thought was correct. When she selected the correct box, after touching a small cross in the middle of the screen, a picture of a salsa dancer flashed on the screen along with a celebratory noise.

The 4 remaining training trials were performed by the participant with guidance from the experimenter in order to solidify the participant's understanding of the task at hand (e.g., choosing the same number). These trials consisted of stimuli that were not pictures but rather dots; the participant was told that these dots were the tops of the dancers' heads. When the participant made the correct selection after touching the cross in the middle of the screen, a picture of a cartoon salsa dancer flashed on the screen with a celebratory noise. If the participant chose incorrectly, a buzz sound occurred and the choice slide remained on the screen until the participant selected the correct choice square. After successfully completing all 4 training trials, the participant was able to move on to the experimental test trials. In these training trials, the child was presented serially with three arrays of 56 dots. There were two comparisons, each presented twice, yielding 4 training trials done by each child (a correct choice option vs. underestimate [56 vs. 20] and a correct choice option vs. overestimate [56 vs. 157]). In this way, the child saw comparisons that largely mirrored what he or she would be exposed to during the test trials, with each trial type probing for the acceptance of a correct outcome relative to an overestimated or underestimated outcome. The correct answer was presented at each spatial location (top, bottom, left, or right) once during the training trials. This was done to ensure that the child did not associate a particular side of the screen with a correct response. These training trial contrasts were chosen to be a relatively easy discrimination (~3:1 ratio of magnitudes).

Experimental trials. The experimental trials were identical to practice trials in their procedure and story, with the sole difference being that no feedback was offered to participants as to whether their answers were right or wrong. Each participant sat approximately 40 cm away from the laptop computer, with the experimenter oriented diagonally behind the participant on these trials to avoid inadvertent cueing of the answers. The participant viewed three priming slides of the same numerosity for 1250 ms each, followed by a set of choice slides presented until the participant indicated his or her choice. The spatial arrangement within each priming set changed between slides even when the number of items remained the same. There were four types of trials in the experimental blocks, and the participant was presented with priming slides of 32–32–32 (50% of the trials) or 96–96–96 (50% of the trials). In the correct versus underestimate trials, the child saw the correct answer contrasted with an underestimated answer. For example, a participant who viewed slides of 32, 32, and 32 would see a choice slide containing 32 dots (the correct answer) and 20 dots (an incorrect underestimated answer). In the correct versus overestimate trials, the participant saw the correct answer contrasted with an overestimated answer (e.g., 32 vs. 52 after being shown 32, 32, and 32 as primes). In the underestimate versus overestimate trials, there was no correct answer displayed. Instead, the participant was shown two incorrect answers, one of which was overestimated and one of which was underestimated (e.g., 20 vs. 52 after seeing 16, 16, and 16 as primes). Finally, in the tie trials, the participant saw two identical numerosities (correct vs. correct). All trials were preceded by a small cross in the middle of the screen, which the participant pressed to orient to a neutral starting point.

The side of the screen showing the correct answer was counterbalanced within participants; half of the time it appeared on the right, and half of the time it appeared on the left. There were 8 experimental trials in the No Order block (2 correct vs. underestimate, 2 correct vs. overestimate, 2 underestimate vs. overestimate, and 2 tie trials). The order of trial presentation within blocks was randomized, as was the absolute placement of objects within each array. The ratio of the incorrect outcomes to the correct outcomes on the choice slide comparisons was 1.6:1 (a relatively challenging, but still discriminable, contrast for children of this age; Halberda & Feigenson, 2008). After completing one block, participants had a brief break before the experiment resumed with the next block.

Order block

Training trials. The training trials for the Order block retained the format of the No Order block; an introductory experimenter-run trial followed by 4 trials in which a correct answer was presented against an incorrect one. The story was slightly different; the experimenter explained that this time a single dancer was dancing by herself, but she was lonely, so she left and soon returned with a partner. The next slide showed two cartoon dancers dancing as a pair. As the experimenter flipped to the next slide, she described how the two dancers left and each returned with a new friend. The third slide showed two pairs of cartoon dancers. The experimenter then asked, “If everyone leaves and comes back with a new friend, which dance floor do you think will be the dance floor at our party?” When the experimenter selected the correct box, a picture of a salsa dancing pair flashed on the screen along with a celebratory noise. In the training trials, the values doubled across the priming sequence (7–14–28), yielding a correct test choice answer that was identical to the No Order value (56). The choices presented after the sequence were identical to those of the No Order training trials (correct choice option vs. underestimate choice [56 vs. 20], correct choice option vs. overestimate choice [56 vs. 157], underestimate choice option vs. correct choice option [20 vs. 56], and overestimate choice option vs. correct choice option [157 vs. 56]).

Experimental trials. The experimental trials for the Order block were identical to those in the No Order block with respect to the type of trials presented (overestimate vs. correct, underestimate vs. correct, overestimate vs. underestimate, and tie trials) and the counterbalancing of the side of screen for the correct answer. However, the Order block had an ordering sequence before the test choices were presented of either 4–8–16 (50% of the trials) or 12–24–48 (50% of the trials). There were also more Order trials overall, with 6 tie trials (2 correct vs. correct, 2 overestimate vs. overestimate, 2 underestimate vs. underestimate), and 4 of each other trial type (overestimate vs. correct, underestimate vs. correct, and overestimate vs. underestimate). See Fig. 1 for a schematic of the experimental trials presented to participants.

Results

Description of analyses and measures

We performed different sets of analyses to test for the estimation biases and spatial displacements that are characteristic of operational momentum. In the first estimation bias analysis (i.e., under conditions of certainty), we analyzed data from the trials in which a correct answer was paired with an incorrect answer at test (the correct vs. overestimate and correct vs. underestimate trials). The measure for this analysis is how frequently the correct answer was chosen when paired with foils that were larger or smaller than the correct answer and how this frequency changed as a function of ordering block. In the second estimation bias analysis (i.e., under conditions of uncertainty), we analyzed data from the trials in which there was high uncertainty; no correct answer was present, and an overestimate was paired with an underestimate at test. The measure for this analysis was how frequently the overestimate was chosen and whether this changed as a function of ordering block. In the spatial biases analysis, we analyzed data from tie trials. Here, the measurement drawn was the frequency of choosing the right-side array and the question of interest was whether this choice of right-side array differed as a function of ordering. For all analyses, each participant’s score was an average of all trials of that particular type (i.e., a participant who chose the overestimate on 2 of the 4 possible trials in which an overestimated outcome was presented alongside the correct outcome would get a score of .50).

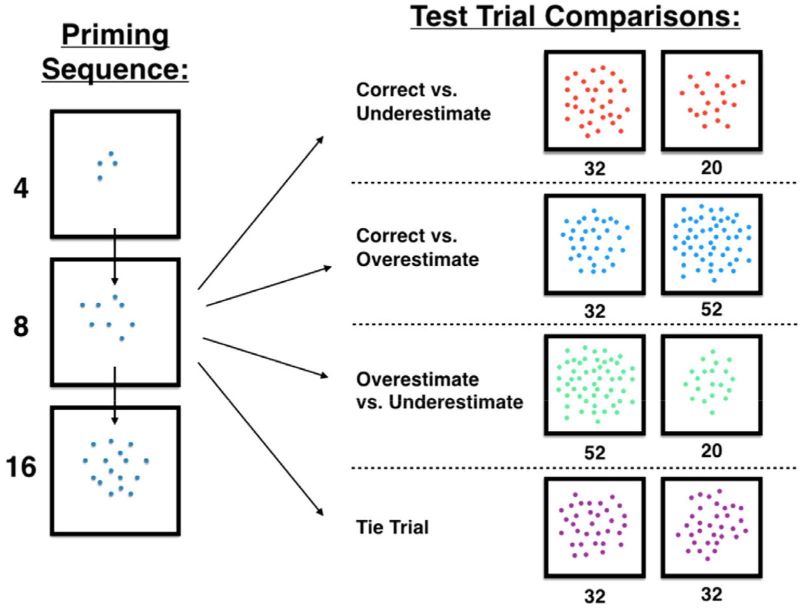

Fig. 1. Schematic illustrating the priming slides and test comparisons for the 4–8–16 Order block trials. Note that for the tie trials, the Order block had three different versions, only one of which is depicted here (correct vs. correct). The other tie trial versions were overestimate versus overestimate and underestimate versus underestimate.

Estimation bias under conditions of certainty

This analysis looked for evidence of estimation bias in trials where there was an objectively correct answer (the correct vs. overestimate trials and the correct vs. underestimate trials). Thus, the measure for this analysis was how frequently the correct answer was chosen when paired with foils that were larger or smaller than the correct answer. In these trials, if participants were experiencing operational momentum, they should be less accurate for the overestimate versus correct trials in the Order block than in the No Order block. For example, a participant who generated an outcome for the 4–8–16 trials of 40 (an overestimate of the correct answer, 32) would state that 52 and 32 are equally good answers. This would not be the case for the No Order block; the participant's generated answer would be centered around 32, with the foil of 52 being readily rejected. Conversely, for the correct versus underestimate trials, if a participant was experiencing operational momentum, he or she should be more accurate in the Order block relative to the No Order block. A participant who generated an outcome of 40 for a 4–8–16 sequence and saw the correct answer of 32 paired with 20 should more easily reject 20 as an answer. Together, this pattern predicted an interaction of the type of trial (correct answer being compared with an underestimate or overestimate) and type of block (whether the participant performed an ordering operation or not). To test this prediction, we calculated the percentage of time participants chose the correct answer and entered it into a repeated-measures analysis of variance (ANOVA) with block (Order or No Order) and trial type (overestimate vs. correct, underestimate vs. correct) as within-participants factors and gender (male or female) as a between-participants factor.

For preschoolers, there was no main effect of block, $F(1, 48) = 0.28$, $p = .60$. Children chose the correct answer 51.3% of the time ($SEM = 2.1$) in the Order block and 49.2% of the time ($SEM = 3.2$) in the No Order block. There was no main effect of trial type, $F(1, 48) = 0.96$, $p = .33$, or gender, $F(1, 48) = 0.05$, $p = .82$. The predicted Block × Trial Type interaction was not present, $F(1, 48) = 0.17$, $p = .68$. There was a significant interaction of gender and trial type, $F(1, 48) = 4.54$, $p = .038$, partial $\eta^2 = .09$. Pairwise comparisons, Bonferroni-corrected for multiple comparisons, indicate that girls were less accurate than boys in the correct versus underestimate trials (39.1%, $SEM = 4.7$ vs. 54.6%, $SEM = 6.0$), $p = .046$. Girls also performed more poorly on the correct versus underestimate trials (39.1%, *SEM* = 4.7) than on the correct versus overestimate trials (60.5%, *SEM* = 5.0), $p = .015$.

For adults, there was a main effect of block, $F(1, 47) = 43.14$, $p < .001$, partial $\eta^2 = .48$, with participants being less accurate in the Order block (70.3%, *SEM* = 2.0) than in the No Order block (87.8%, *SEM* = 2.1). There was also a main effect of trial type, $F(1, 47) = 23.27$, $p < .001$, partial $\eta^2 = .33$. Adults were overall more accurate in the overestimate versus correct trials (87.5%, *SEM* = 1.9) than in the underestimate versus correct trials (70.6%, *SEM* = 2.7). We did not find the predicted Block × Trial interaction, $F(1, 47) = 0.15$, $p = .70$, that would indicate shifting overestimation or underestimation as a function of an ordering operation. There was no significant effect of gender, $F(1, 47) = 1.14$, $p = .29$.

Estimation bias under conditions of uncertainty

Previous work on operational momentum for addition and subtraction indicates that this bias is heightened under conditions of estimation uncertainty (Charras, Brod, & Lupiáñez, 2012; Knops, Viarouge, et al., 2009; McCrink et al., 2007). The evaluation of whether there was an estimation bias under conditions of uncertainty applied most clearly to the trials in which the overestimated amount was contrasted with the underestimated amount; here, there was no correct answer presented. If participants were experiencing operational momentum after an ordering operation, they should be more likely to choose the overestimated amount in the Order block than in the No Order block. To test this, we calculated how frequently the overestimate was chosen when an overestimate and an underestimate were presented as test choices. The percentage of time participants chose the overestimate was calculated and placed into a repeated-measures ANOVA with block type (Order or No Order) as a within-participants factor and gender (male or female) as a between-participants factor.

For preschoolers, there was no main effect of block, $F(1, 48) = 0.01$, $p = .98$; children were equally likely to choose the overestimate in the Order block (47.9%, *SEM* = 4.9) and in the No Order block (48.1%, *SEM* = 5.8). There was no main effect of, or interactions with, gender (see Fig. 2). For adults, there was a main effect of block, $F(1, 47) = 8.03$, $p = .007$, partial $\eta^2 = .06$; adults were more likely to choose the overestimate in the Order block (44.9%, *SEM* = 4.1) than in the No Order block (26.4%, *SEM* = 5.1) (see Fig. 2). There was no main effect of, or interactions with, gender.

Spatial biases

The evaluation of whether there were spatial biases prompted by the ordering operation pertains primarily to the tie trials, in which there was no magnitude information that could influence participants' choice (because both sides of the screen displayed the same value). If participants had a general bias to attend to the right side of space after computing ascension values, they should be more likely to indicate that right-side answers were more acceptable than left-side answers for the Order block (relative to the No Order block). To test this prediction, participants' percentage choice of right-side answers was placed into a repeated-measures ANOVA with block type (Order or No Order) as a within-participants factor and gender (male or female) as a between-participants factor. To reiterate, the measurement here was the frequency of choosing the right-side array for each type of trial.

For preschoolers, there was a significant main effect of block type, $F(1, 48) = 4.32$, $p = .04$, partial $\eta^2 = .08$. For these tie trials, children chose the right-side answer more in the Order block (64.4%, *SEM* = 4.4) than in the No Order block (49.3%, *SEM* = 5.8). For adults, there was a significant main effect of block type, $F(1, 47) = 4.09$, $p = .048$, partial $\eta^2 = .08$. For these tie trials, right-side answers were chosen more frequently in the Order block (56.1%, *SEM* = 3.6) than in the No Order block (45.2%, *SEM* = 4.7) (see Fig. 3).

Exploratory analysis: Examining high-performing preschoolers

The data from the preschoolers indicate that they did not do very well overall in the task. The question arises as to whether children would look more like adults if they were more competent at the task overall. Here, we grouped children into low performers (<50%; $n = 12$), mid-performers (50%; $n = 23$), and high performers (>50%; $n = 15$). Because there is a general bias to underestimate large sets of

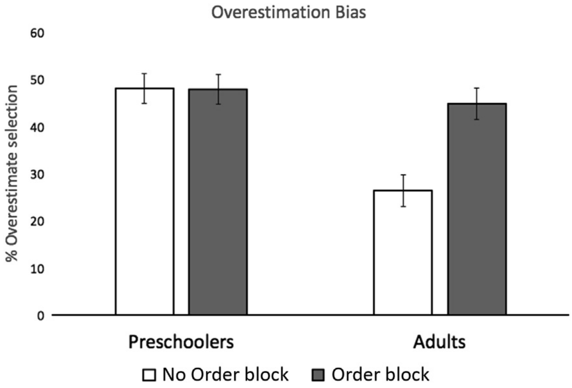

Fig. 2. Percentage of overestimate choice by preschoolers and adults, as a function of block (Order or No Order), for those trials in which an overestimate was contrasted with an underestimate (i.e., estimation uncertainty). Within-participants error bars indicate ±1 standard error of the mean (calculated via the method described in Cousineau, 2005).

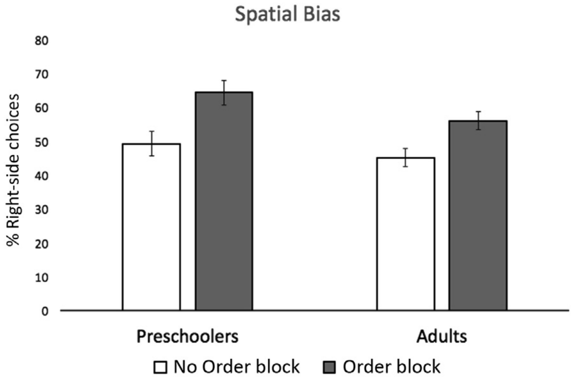

Fig. 3. Percentage of right-side choices by preschoolers and adults, as a function of block (Order or No Order), for trials in which the values presented on the left and right sides of the screen were identical (i.e., tie trials). Within-participants error bars indicate ±1 standard error of the mean (calculated via the method described by Cousineau, 2005).

objects (Izard & Dehaene, 2008), it is difficult to define chance performance. A large population of children were at 50% exactly, and this striation allows us to have roughly equal *n*s in the three groups. In this way, we could pull out the children with varying levels of performance when a correct answer *was* present and analyze them to see whether they varied with respect to their operational momentum biases for those trials in which a correct answer was *not* present. A series of paired-samples *t* tests was performed for these populations and Bonferroni-corrected for multiple comparisons. Low performers showed no right-side bias for Order block over No Order block in the overestimate versus underestimate trials (64.6%, *SEM* = 6.6 vs. 58.3%, *SEM* = 8.4), *p* = .55, or the tie trials (62.5%, *SEM* = 9.0 vs. 62.5%, *SEM* = 11.2), *p* = 1.00. Mid-performers also showed no right-side bias for Order block over No Order block in the overestimate versus underestimate trials (60.9%, *SEM* = 4.8 vs. 58.7%, *SEM* = 6.1), *p* = .77, or in the tie trials (58.7%, *SEM* = 5.9 vs. 56.5%, *SEM* = 8.1), *p* = .84. High performers did not show a significant right-side preference for Order block over No Order block in the overestimate versus underestimate trials (56.7%, *SEM* = 5.9 vs. 46.7%, *SEM* = 7.5), *p* = .29, but this effect did emerge in the tie trials (68.8%, *SEM* = 8.0 vs. 33.3%, *SEM* = 10.0), *p* = .009.

Exploratory analysis: Detecting age effects

One additional question of interest was whether the strength of the effects found differed as a function of whether the participants were adults or children. To address this question in the context of overestimation effects, we looked further at participants' tendency to choose an overestimate when comparing an overestimate with an underestimate in the Order and No Order blocks. These scores were entered into a repeated-measures ANOVA with block (Order or No Order) as a within-participants factor and age (child or adult) as a between-participants factor. There was a significant effect of age, $F(1,\ 97) = 4.49$, $p = .04$, partial $\eta^2 = .04$, with children being generally more likely to choose the overestimate than adults. There was an interaction between block and age, $F(1,\ 97) = 4.03$, $p = .047$, partial $\eta^2 = .04$, with adults, but not children, choosing the overestimate more in the Order block relative to the No Order block. To address the question of age differences in the context of spatial bias effects, we examined right-side choices for the tie trials in the Order and No Order blocks. These scores were entered into a repeated-measures ANOVA with block (Order or No Order) as a within-participants factor and age (child or adult) as a between-participants factor. There was a main effect of block, $F(1,\ 97) = 6.32$, $p = .01$, partial $\eta^2 = .06$, with right-side choices being made more frequently in the Order block than in the No Order block. There was no significant interaction with age, $F(1,\ 97) = 0.01$, $p = .96$.

Discussion

This study investigated operational momentum in preschoolers and adults during ordering. To date, the evidence for this spatial–numerical phenomenon in childhood is mixed and has been examined exclusively for addition and subtraction operations (Knops et al., 2013). Results showed that adults, but not preschoolers, exhibited a greater tendency to overestimate the next magnitude in a series of magnitudes when performing an ascending operation compared with simply estimating a magnitude. Importantly, this was the case only in the face of uncertainty (during overestimate vs. underestimate trials). Both adults and preschoolers selected a right-side answer more frequently when performing an ascending operation compared with a simple estimation task. Thus, we found some evidence for developmental continuity in the presence of lateralized spatial associations during the operation of ordering. However, in spite of the fact that both adults and children exhibit the spatial biases characteristic of operational momentum, only adults exhibited the predicted overestimation bias (and even then, only under conditions of estimation uncertainty).

In some respects, the lack of overestimation after ordering in the preschoolers is surprising. Not only is this finding distinct from the direct adult comparison within the study, but it also differs from findings on operational momentum in infancy. Macchi Cassia et al. (2017) tested 4-month-old infants in a version of an ordering task similar to that used here. Recall that in this study infants were habituated to ascending or descending series of magnitudes and then shown test sequences of ascending or descending magnitudes that were smaller or larger than the habituated sequences. Infants who habituated to ascending series looked longer, on average, to a test ascending series that was less numerous than what they had learned (and the opposite was found for infants habituated to descending sequences). This pattern is thought to reflect a novelty response prompted by the generation and acclimation of overestimates during the ordering task. Thus, although there is evidence that infants generate mis-estimates when ordering magnitudes, the current study reveals that preschoolers do not.

In other respects, however, the current finding of a lack of overestimation after ordering in the preschoolers accords with other work on operational momentum in childhood. Knops et al. (2013) tested adults and young elementary school children for operational momentum during addition and subtraction. As in this study, these children did not show mis-estimation effects as a group (e.g., overestimating addition problems, underestimating subtraction problems), but the adults did. In a separate visual spatial attention task that was part of the same study, children were tested on their ability to overcome an invalid cue when shifting attention (Knops et al., 2013). Tellingly, the degree to which the children exhibited adult-like spatial attention patterns was correlated with their degree of exhibited operational momentum (Knops et al., 2013).[1] This led Knops et al. to suggest that it is the maturation of the spatial attention networks that leads to shifting of spatial attention along the number line when operating.

Given that current theories posit that it is the spatial shifts of attention along an internally represented horizontal continuum that prompts miscalculation of arithmetic outcomes (cf. McCrink & Hubbard, 2017), how can we reconcile the presence of one without the other? One line of interpretation stems from the methodology of the current study. It is possible that there were indeed miscalculations arising from spatial shifts in the preschoolers, similar to the infants, but the study's design dampened them for the preschoolers. For instance, in the infant study (Macchi Cassia et al., 2017), an infant-controlled habituation procedure was employed, which gave each single participant ample time to encode and form a robust representation of the ordering operation. Moreover, during habituation infants were given multiple examples of the same ordering rule. In the current study, children instead saw just one example sequence before being probed for their test choice, and presentation times were fixed and rather short. Thus, it is reasonable to assume that the representation of the ordering rule formed by the preschoolers was not as strong as that formed by the infants; as a result, the spatial shifts of attention might have been present but too weak to actually influence their calculations. In addition, the infants in Macchi Cassia et al. (2017) study received ordered *sequences* (e.g., three ordered slides) at test, whereas the preschoolers in the current study received two quantities from which to choose an answer. This difference in the two testing paradigms may have resulted in a more apparent match between the priming/habituation ordering operation and the test operation for the infants compared with that for the preschoolers, which in turn increased operational momentum for the infants.

Another possibility is that there were genuinely no miscalculation biases in children because they did not fully grasp the task. Indeed, overall children performed poorly on the task. It follows logically that if one cannot grasp the task, then one will not generate a correct outcome to begin with and, furthermore, will not experience an added overestimation error from operational momentum. To intuit the ordinal relationship between the amounts, children needed to very quickly learn one type of relation for one block of trials (i.e., either estimate the same amount for the No Order block or double the last value in the sequence for the Order block) and then abandon that relation for the second block of trials. However, this performance account seems to be unlikely if one examines data from the higher-performing preschoolers in the study. This population was able to reliably choose the correct answer when it was present and showed spatial shifts in their responses in accord with operational momentum. Yet, even the higher-performing preschoolers did not show a stronger tendency to overestimate after an ordering task compared with an estimation task.

The second line of interpretation for why there are stronger spatial biases than overestimation effects is theoretical. Perhaps, for young children, the emergence of operational momentum reflects a protracted developmental entwining of visuospatial attention and mental calculations. As the findings from Knops et al. (2013) suggest, the spatial and mis-estimation effects of operational momentum may emerge only in the context of a fully developed attentional system and a strong interface between visuospatial attention and the mental number line. Pinheiro-Chagas, Didino, Haase, Wood, and Knops (2018) recently found no evidence of operational momentum among 8-year-olds but found an emergent operational momentum effect during addition and subtraction among children aged 9–12 years. Perhaps, as visuospatial attention systems become more adult-like, these shifts of attention begin to interface with the number line and this leads to operational momentum emerging in older children. Given that addition/subtraction and ordering both are arithmetic operations, it is possible that the

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[1] To investigate whether that is the case here, we calculated a difference score for how often participants chose the right-side answer in the Order block versus the No Order block for the tie trials. This gave us a measure of spatial displacement generated by the operation for each participant. We also calculated a difference score for how often participants chose the overestimated amount in the Order block versus the No Order block for the overestimate versus underestimate comparison. For both adults and children, there was no relation between the strength of spatial displacement tendency and overestimation tendency. However, we must be cautious in drawing conclusions from this analysis because it is exploratory and the experimental design was not crafted with it in mind (e.g., uneven numbers of trials in the Order and No Order blocks).

ordering operations investigated in the current study also follow a protracted developmental trajectory.

How, then, do we reconcile this account—a protracted interface of spatial attention and the mental number line with spatial shifts emerging before estimation biases—with the findings on operational momentum in infancy? For both ordering operational momentum and addition/subtraction operational momentum, there is evidence that infants are miscalculating outcomes in accord with the "direction" of the arithmetic operation (Macchi Cassia et al., 2017; McCrink & Wynn, 2009). Yet, this bias is not apparent in childhood (Knops et al., 2013; Pinheiro-Chagas et al., 2018; current study). One possibility is that the underlying construct of associations between magnitude and space in an infant's mind is qualitatively different from that in an adult's mind, and early childhood is when this shift occurs (see McCrink & de Hevia, 2018, for a detailed version of this account). There has been a recent spate of data strongly suggesting that even newborns and nonhuman animals possess some sort of mental number line, with small quantities being associated with the left side and large quantities with the right side (Bulf et al., 2016; de Hevia et al., 2017; Rugani, Vallortigara, Priftis, & Regolin, 2015). At the same time, there are findings from preschool age and older that children's associations between magnitudes and lateral space are sensitive to the spatial customs of their environment (Opfer & Furlong, 2011; Goebel et al., 2018; Shaki et al., 2012). Toddlerhood and very early childhood, then, must logically be a transitional period in which associations between magnitude and space move from an innate construct to one that is culturally sensitive and mature. Part of this maturity involves a rebuilding of the spatial associations to reflect the spatial biases of children's culture and then the integration of this mature representation with a slowly developing visuospatial attention system. Put simply, we may observe differing findings on operational momentum in infancy and childhood because the underlying basis of operational momentum—associations between magnitude and space—differs meaningfully between infants and children.

Beyond the issue of differences in spatial and calculation operational momentum, one striking aspect to the current results is the degree of underestimation found throughout the study and for both adults and children. Although we observed that adults are more likely to choose an overestimated outcome after ordering (compared with estimating), under conditions of certainty (i.e., when the correct amount was available) both populations generally avoided choosing the larger test amount throughout. Both children and adults were less accurate for the underestimated versus correct trials than for the overestimated versus correct trials. That is, participants were systematically more likely to select underestimated outcomes relative to correct responses compared with overestimated outcomes relative to correct responses. This pattern is not without precedent in the literature. Izard and Dehaene (2008) found that when observers were asked to estimate the approximate numerosity of dot arrays, in the absence of calibration they most often underestimated. Classic studies on numerosity estimation also have found that participants are poor at spontaneously approximating nonsymbolic numerosities (Krueger, 1982, 1984; Minturn & Reese, 1951). For example, Minturn and Reese (1951; as cited in Izard & Dehaene, 2008) reported responses diverging from the true numerosity by a factor as high as 4 (e.g., responses range from 50 to 700 for a stimulus containing 200 dots). Importantly, these studies involved adults viewing dot arrays and then generating a number word without counting, whereas the current study did not involve number words at all. With regard to the current study, there was clearly a tendency to underestimate for all participants, which may or may not be related to the same phenomenon in the symbolic-to-nonsymbolic mapping literature. This tendency dampened the overall results within the Order block and required us to focus on comparisons between the Order block and the No Order block.

Another notable aspect to the current findings is that, insofar as we observed the phenomenon of operational momentum here, we saw it only under conditions of uncertainty. When the same magnitude was presented at test on either side of the screen, or both an overestimate and an underestimate were presented together, miscalculation and spatial biases characteristic of operational momentum emerged. This result supports the idea that operational momentum is at least partially a type of spatial heuristic: when adding, ordering, or multiplying, we generate a logical rule of "accept more than the initial amount" and, furthermore, this concept of "more" is associated with right side. For adults, at least, this rule is not the only concept in play given that it has been observed that they generate arithmetic addition outcomes that skew just slightly more than the correct answers and successfully reject outcomes that are much larger than the correct answers (Knops, Viarouge, et al., 2009; McCrink et al., 2007). Furthermore, Fischer and colleagues have delineated additional types of heuristics that can lead to operational momentum such as anchoring on a particular starting value for an equation and even just seeing the symbolic operation sign + (plus) or – (minus) (Pinhas & Fischer, 2008; Pinhas, Shaki, & Fischer, 2014). For adults, operational momentum is heightened under cognitive load, as is also the case for other non-arithmetic logical heuristics (Kahneman & Tversky, 2013). For example, operational momentum effects are heightened when participants are operating over larger, more imprecise values (Knops, Viarouge, et al., 2009), when participants are under time constraints and cannot accurately count (McCrink et al., 2007), or when participants' attention is drawn to a distractor task (McCrink & Hubbard, 2017). The implementation of the tie trials and overestimated versus underestimated trials in the current study elicited operational momentum in a way that conditions of more certainty did not. This provides additional evidence for a spatial heuristics account of operational momentum and establishes that these logical–spatial heuristics are present as early as the preschool years.

Going forward, future studies should elaborate on the current design and examine operational momentum in a descending ordering task as a comparison point for the ascending ordering operation offered here. Given the constraints of such a young population (with children as young as 37 months), the adoption of a between-participants design would have proved necessary to include a descending operation because such young children lose interest in the task rather quickly. However, the infants tested in Macchi Cassia et al. (2017) showed an underestimation tendency after habituation to descending numerical sequences, which was not dissimilar to the overestimation tendency they showed after habituation to ascending sequences. Therefore, testing for the generalizability of the obtained findings to descending ordering operations in young children would be worth further investigation. Second, researchers should explore how calibration may affect operational momentum. For instance, Izard and Dehaene (2008) found that presentation of a concrete reference number prior to estimation enabled participants to approximate numerosities more precisely. Whereas the current study did not involve number words and involved only nonsymbolic quantities, future studies could extend this work by examining how calibration may affect operational momentum during ascending and descending ordering. In addition, the current study used a sample of children from a largely English-speaking country. Future work could examine populations in other countries such as Israel and Iran, whose languages moderate the direction of the mental number line early in childhood (Shaki et al., 2012).

It is important to note that, in this experiment, we did not control for spatial extent cues that covary with numerical magnitude. As the number of items in each array increased, so did the area, perimeter, density, and convex hull of the set. Thus, the conclusions that we can draw from this design are limited to that of relations between a lateralized spatial continuum and quantity. The quantity representations here—what we and many others casually call "number" in the context of spatial–numerical relations—are computed over bounded discrete objects, but they ultimately may be underlain by either spatial processing *or* abstract numerical processing. That is, the ordering operation prompting this mental "momentum" may be performed by participants over spatial magnitudes (e.g., Macchi Cassia et al., 2016) or abstract numerical magnitudes (e.g., McCrink et al., 2007). Given that Macchi Cassia et al. (2017) found that, with infants, operational momentum for ordering arose only when both spatial and numerical characteristics were ordered, we believed that the first investigation of ordering operational momentum with children and adults should preserve both these characteristics of magnitude representations. Future experiments on this topic should address the question of whether ordering operational momentum in these populations is truly a spatial–numerical interaction or rather a spatial–magnitude interaction (via careful stimulus controls or cross-modal methods that circumvent visual stimuli confounds).

In summary, the results of the current study suggest that both adults and preschool children tap into magnitude–space associations to support their ordering calculations. The fact that this occurs with preschoolers, who have yet to enter formal schooling, implies that this mechanism arises without instruction and is part of an intuitive logical system that capitalizes on early developing links between spatial structure and quantity representations. Using a spatial scaffold to represent number is an important part of enriching mathematical cognition (Gunderson, Ramirez, Beilock, & Levine, 2012) and has implications for the etiology of dyscalculia and typically developing children's developmental trajectories in formal math (Huber, Sury, Moeller, Rubinsten, & Nuerk, 2015). The current child-friendly paradigm offers a promising avenue to further explore the development of the phenomenon of operational momentum, and the findings presented here lead to novel predictions related to the relationship of space, magnitude, and visuospatial attention throughout development.

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